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Stability of Menisci in Detached Bridgman Growth

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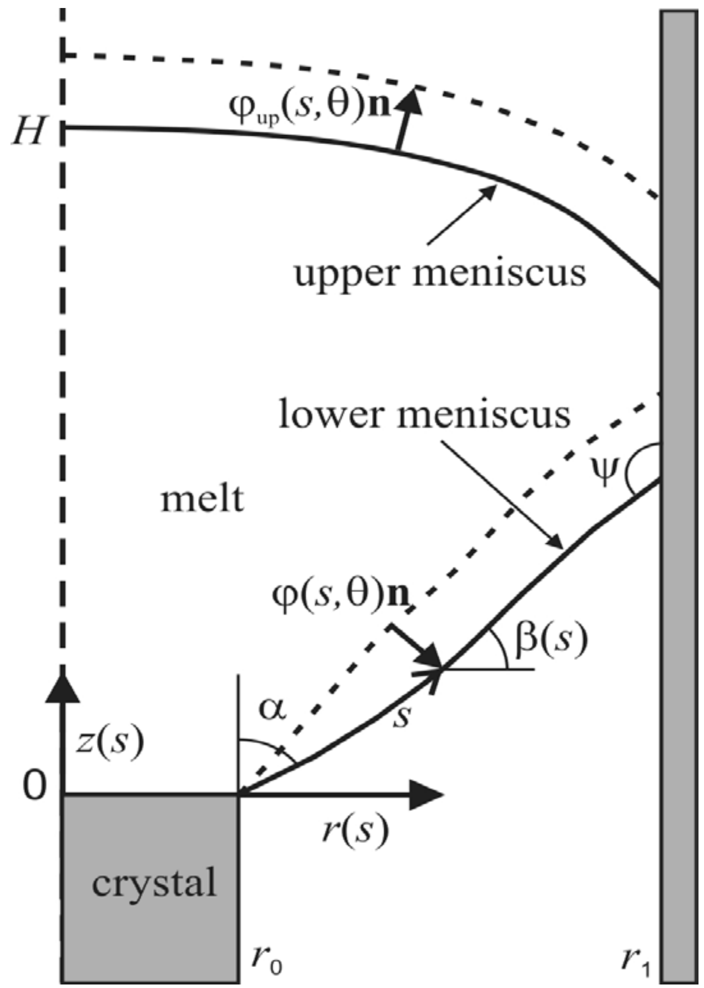
schematic



H
φ_up(s,θ)n
upper meniscus
lower meniscus
ψ
melt
φ(s,θ)n
β(s)
α
s
z(s)
0
r(s)
crystal
r_0
r_1

Energy

Free energy of the system depends on melt surfaces

$$U = -p_{up}V_{up} - p_d V_d + \sigma S_{up} + \sigma S_d + \sigma_{gc}\Sigma_{gc} + \sigma_{fc}\Sigma_{fc} + g\rho\left(\int z dV\right)$$

σ - melt surface tension
σ_{gc} – gas - crucible interface tension
σ_{fc} – melt- crucible interface tension
ρ – melt density, g – gravity acceleration
p_{up} , p_d – upper and lower pressures

energy variations

Energy depends on surface shapes

$$U = U_0 + \delta U + 1/2\,\delta^2 U + \dots$$

Extremum of energy yields the Young-Laplace equation

$$\delta U = 0 \quad \rightarrow \quad \sigma\left(k_1 + k_2\right) = \rho\phi + c,$$

$$F = -\nabla\phi, \quad \phi = gz - \omega^2 r^2/2$$

k_1+k_2 – sum of principal curvatures

ω - angular frequency of crucible with melt

energy variations

Energy minimum –

second variation of energy has to be positive

$$\delta^2 U > 0$$

$\delta^2 U = 0$ defines neutral stability boundaries

1. Upper and lower menisci are axisymmetric
2. Melt volume is preserved
3. Menisci perturbations are non-axisymmetric, in general.

non-dimensional numbers

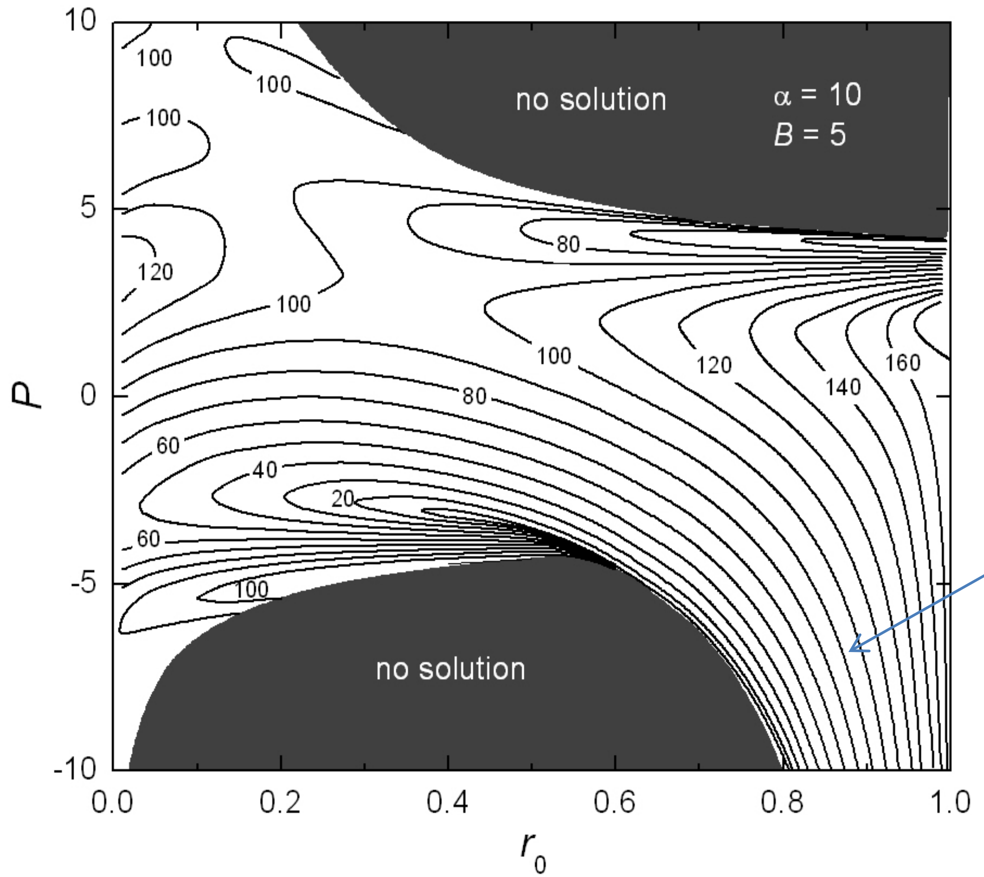
$$B = \frac{g\rho r_1^2}{\sigma}$$ - Bond number

$$P = \frac{\left(p_{up} - p_d + g\rho H - 2\kappa\sigma\right) r_1}{\sigma}$$ - differential pressure parameter

$$T_{up} = \frac{p_{up} r_1^4}{\sigma V_{up}}, \quad T_d = \frac{p_d r_1^4}{\sigma V_d}$$ - finite gas volume parameters

$$W = \frac{\rho\omega^2 r_1^3}{2\sigma}$$ - Weber number

Young – Laplace equation solution

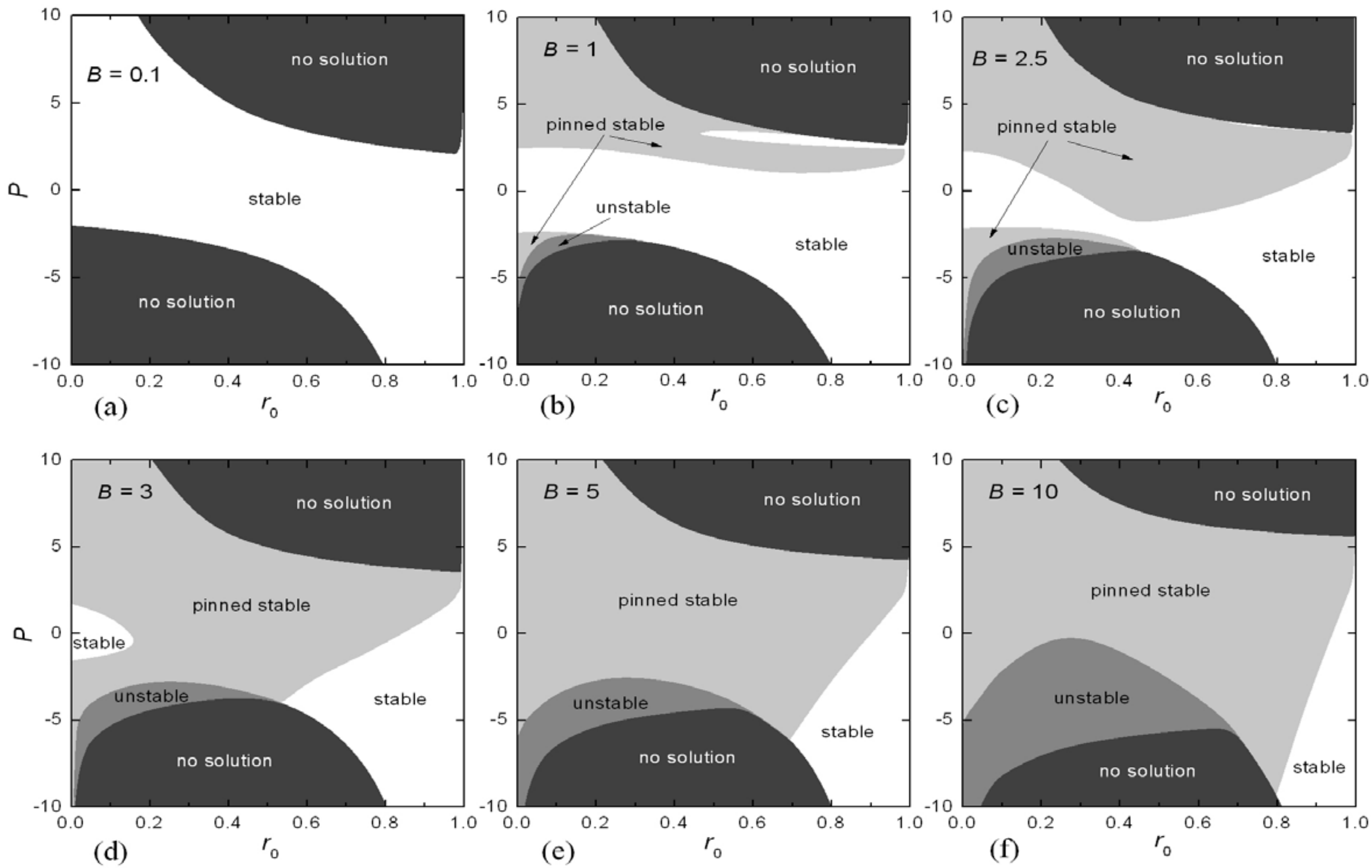


Contact angles of menisci with the crucible wall

equi-contact angle lines

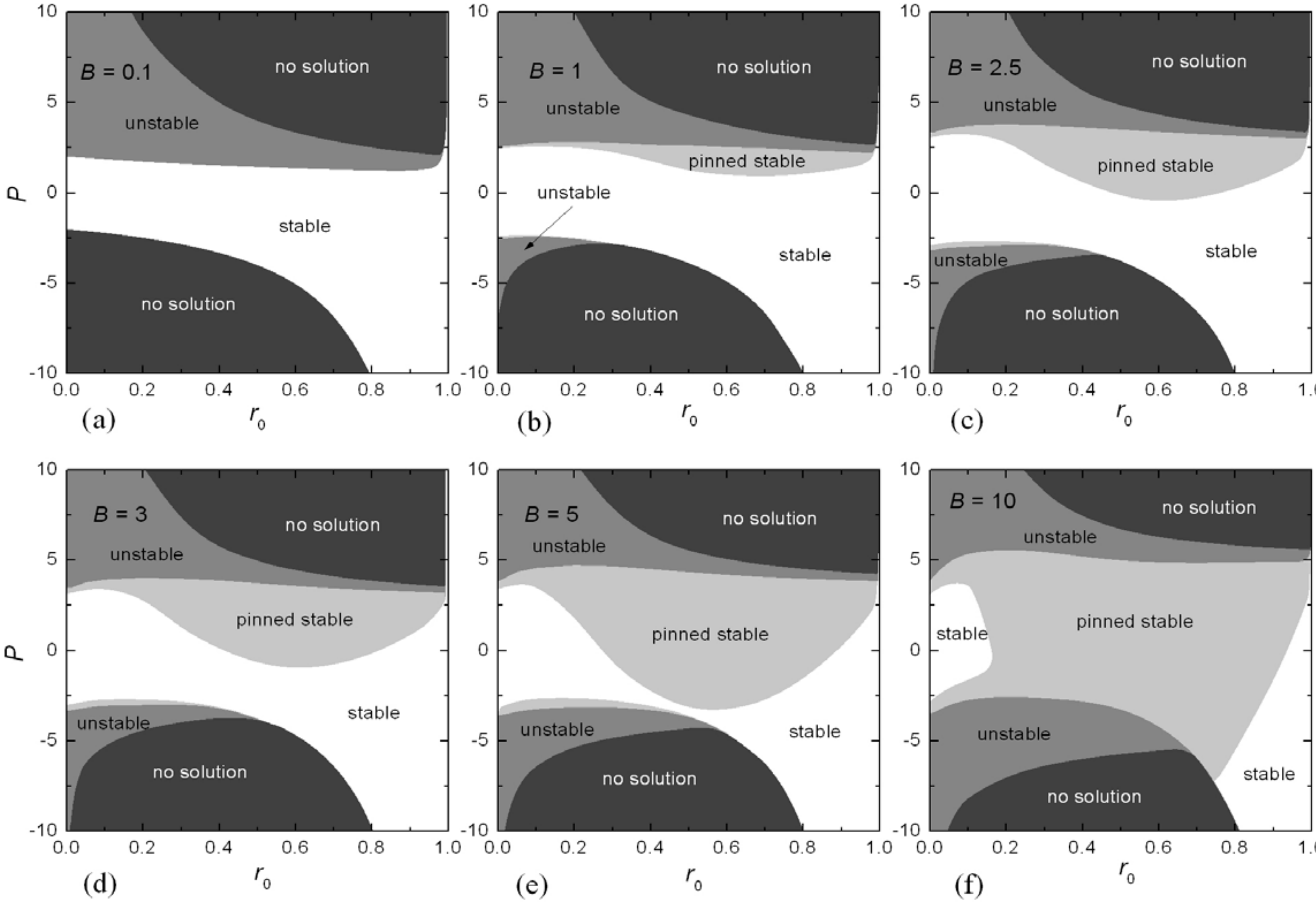
Stability maps
non-axisymmetric perturbations

$\alpha_{gr}=10^0$



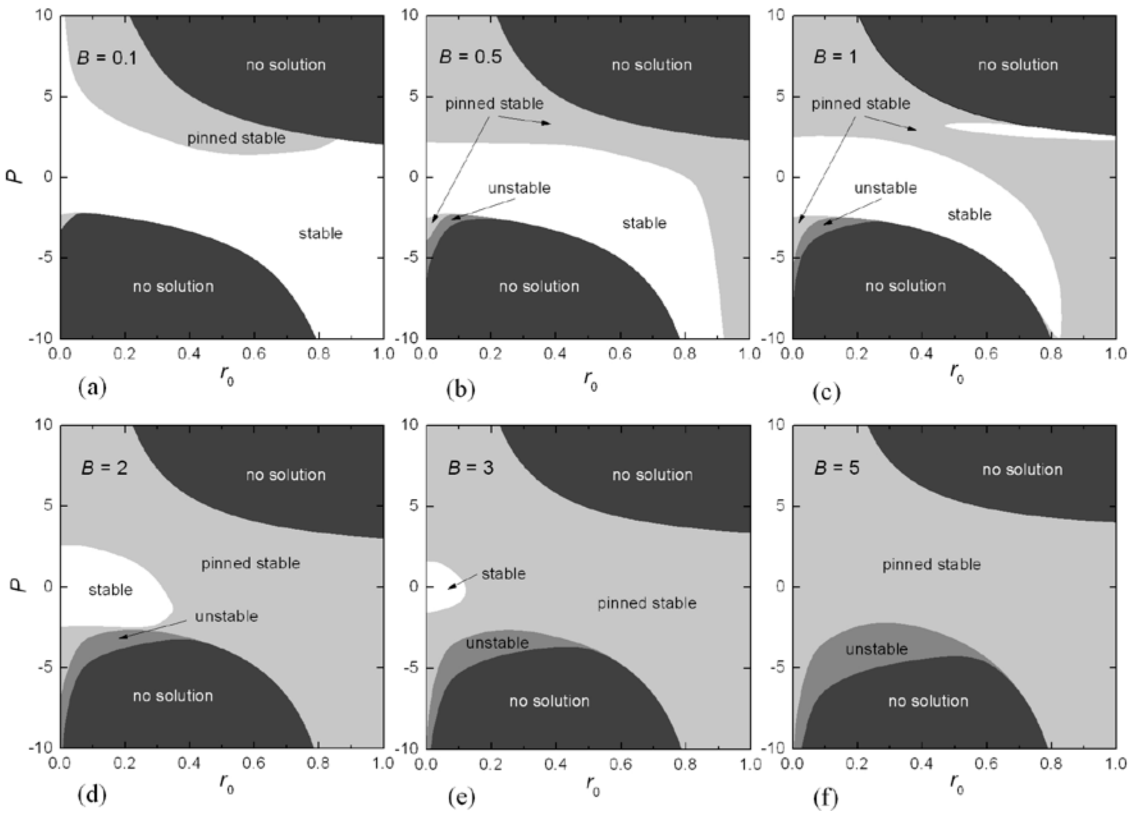
Stability maps
axisymmetric perturbations

$\alpha_{gr}=10^0$

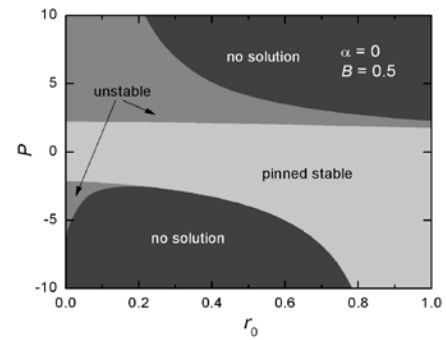


Stability maps
axisymmetric perturbations

$\alpha_{gr}=0^0$



Non-axisymmetric modes

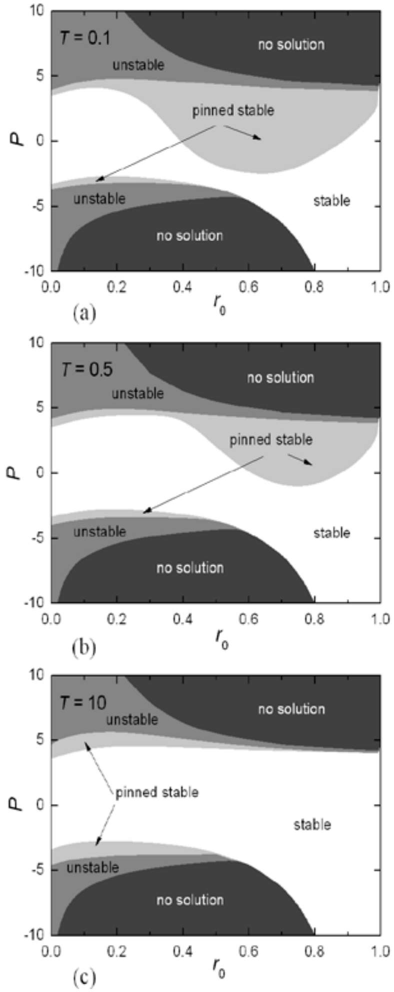


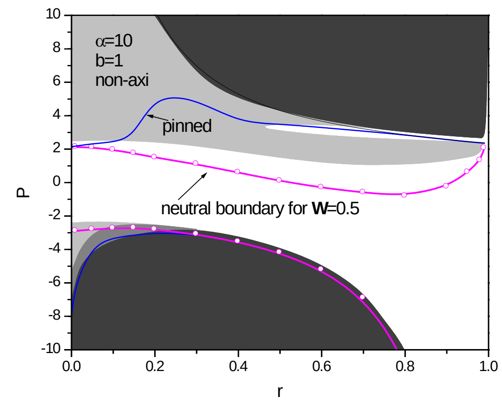
Axisymmetric modes

Effects of rotation and finite gas volumes

$\alpha_{gr}=10^0$

axisymmetric modes





Rotation diminishes stability
of only non-axisymmetric modes

Finite volumes enhances
stability of axi-symmetric modes